(2) $$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$$

(3) $$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

(4) $$\int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left| x + \sqrt{x^2 - a^2} \right| + C$$

(5) $$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$$

(6) $$\int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left| x + \sqrt{x^2 + a^2} \right| + C$$

We now prove the above results:

(1) We have $\dfrac{1}{x^2 - a^2} = \dfrac{1}{(x-a)(x+a)}$

$$= \frac{1}{2a} \left[\frac{(x+a) - (x-a)}{(x-a)(x+a)} \right] = \frac{1}{2a} \left[\frac{1}{x-a} - \frac{1}{x+a} \right]$$

Therefore, $$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \left[\int \frac{dx}{x-a} - \int \frac{dx}{x+a} \right]$$

$$= \frac{1}{2a} \left[\log |(x-a)| - \log |(x+a)| \right] + C$$

$$= \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

(2) In view of (1) above, we have

$$\frac{1}{a^2 - x^2} = \frac{1}{2a} \left[\frac{(a+x) + (a-x)}{(a+x)(a-x)} \right] = \frac{1}{2a} \left[\frac{1}{a-x} + \frac{1}{a+x} \right]$$