$\Rightarrow Pp + Qq = R$

where $P = \left(\frac{\partial u}{\partial y}\frac{\partial v}{\partial z} - \frac{\partial u}{\partial z}\frac{\partial v}{\partial y}\right)$

$Q = \frac{\partial u}{\partial z}\frac{\partial v}{\partial x} - \frac{\partial u}{\partial x}\frac{\partial v}{\partial z}$

$R = \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}$.

Which is of the same form of eqn (1)

$\therefore$ (3) is g.s. of (1).

Now consider $u(x,y,z) = c_1$ & $v(x,y,z) = c_2$

where $c_1$ & $c_2$ are arbitrary constants.

By differentiating, we get

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz = 0$$

$$\Rightarrow \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz = 0 \quad \text{---(i)}$$

and $$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz = 0$$

$$\Rightarrow \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz = 0 \quad \text{---(ii)}$$

By cross multiplication we get

$$\frac{dx}{\frac{\partial u}{\partial y}\frac{\partial v}{\partial z} - \frac{\partial u}{\partial z}\frac{\partial v}{\partial y}} = \frac{dy}{\frac{\partial u}{\partial z}\frac{\partial v}{\partial x} - \frac{\partial u}{\partial x}\frac{\partial v}{\partial z}} = \frac{dz}{\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}}$$

$$\Rightarrow \frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

which is same as the eqn (4)

$\therefore u(x,y,z) = c_1$ & $v(x,y,z) = c_2$ are solutions of (4).

**Note:** Equations (4) are called Lagrange's auxiliary eqns (or) subsidiary eqns for (1).

$\rightarrow$ **working Rule for solving of Lagrange's eqn $Pp + Qq = R$:**

**Step 1:** Write the given eqn in standard form $Pp + Qq = R$ ---(1)

**Step 2:** Write the Lagrange's auxiliary eqns for (1), namely $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ ---(2)