Note!

The converse of the above need not be true. i.e f is continuous and possesses partial derivative at a point then f need not be differentiable at that point.

For example:-

$$f(x,y) = \begin{cases} \dfrac{x^3-y^3}{x^2+y^2} & ; (x,y) \neq (0,0) \\ 0 & ; (x,y) = (0,0) \end{cases}$$

is continuous and possesses partial derivatives but is not differentiable at the origin.

Sol put $x = r\cos\theta$, $y = r\sin\theta$

$$\therefore \left| \frac{x^3-y^3}{x^2+y^2} - 0 \right| = \left| r(\cos^3\theta - \sin^3\theta) \right|$$

$$\leq |r| \left[|\cos^3\theta| + |\sin^3\theta| \right]$$

$$\leq |r| [1+1]$$

$$= 2|r|$$

$$= 2\sqrt{x^2+y^2} < \epsilon$$

whenever $x^2 < \frac{\epsilon^2}{8}$, $y^2 < \frac{\epsilon^2}{8}$

(or)

$|x| < \frac{\epsilon}{2\sqrt{2}}$, $|y| < \frac{\epsilon}{2\sqrt{2}}$

$\therefore \left| \frac{x^3-y^3}{x^2+y^2} - 0 \right| < \epsilon$ whenever $|x| < \frac{\epsilon}{2\sqrt{2}}$, $|y| < \frac{\epsilon}{2\sqrt{2}}$

$$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x^2+y^2} = 0$$

$\therefore f(x,y)$ is conti. at $(0,0)$.