3.(b) The ellipsoid with equation $x^2+2y^2+z^2=4$ is heated so that its temperature at (x,y,z) is given by $T(x,y,z)=70+10(x+z)$. Find the hottest and coldest points on the ellipsoid.

Sol'n: Given that the ellipsoid $x^2+2y^2+z^2=4$ —— (1) is heated so that its temperature at a point (x,y,z) is given by

$$T(x,y,z)=70+10(x+z) \quad \text{——} (2)$$

we have to find the hottest and coldest points on the ellipsoid.

Now, by using Lagranges multiplier method.

Consider the function

$$F=70+10(x+z)+\lambda(x^2+y^2+z^2-4)$$

$$dF=[10+2x\lambda]dx+4y\lambda dy+(10+2z\lambda)dz$$

For Stationary values $F_x=F_y=F_z=0$.

$$\Rightarrow 10+2x\lambda=0 \quad \text{——} (3)$$

$$4y\lambda=0$$

$$10+2z\lambda=0 \quad \text{——} (4)$$

Now $10+2x\lambda=0 \Rightarrow \lambda\neq 0$

and so $0=4y\lambda \Rightarrow y=0$.

from (3) & (4), we have

$$-2x\lambda=-2z\lambda$$

$$\Rightarrow x=z$$

Substituting $x=z$ & $y=0$ in equation (1), we have