$$\frac{1}{x}+\lambda x=0 \quad , \quad \frac{1}{y}+\lambda y=0 \, , \quad \frac{1}{z}+\lambda z=0$$

$$\Rightarrow -\frac{1}{\lambda}=x^2=y^2=z^2 \quad \text{(or)} \quad x=y=z$$

Thus V is stationary when $x=y=z=a/\sqrt{3}$, from (2)

The lengths of the edges of the rectangular Parallelopiped are $2x, 2y, 2z$. So V is stationary when the rectangular parallelopiped is a cube.

Now regard x and y as independent variables and z as a function of x and y given by (2).

From (1), $\log V=\log 8+\log x+\log y+\log z$

$$\therefore \frac{1}{V}\frac{\partial V}{\partial x}=\frac{1}{x}+\frac{1}{z}\cdot\frac{\partial z}{\partial x}$$

Differentiating (2) partially w.r.t x taking y as constant, we get $2x+2z\left(\frac{\partial z}{\partial x}\right)=0 \Rightarrow \frac{\partial z}{\partial x}=-x/z$.

$$\therefore \frac{1}{V}\frac{\partial V}{\partial x}=\frac{1}{x}+\frac{1}{z}\cdot\frac{-x}{z}=\frac{1}{x}-\frac{x}{z^2}$$

So that
$$\frac{1}{V}\frac{\partial^2 V}{\partial x^2}-\frac{1}{V^2}\left(\frac{\partial V}{\partial x}\right)^2=-\frac{1}{x^2}-\frac{1}{z^2}+\frac{2x}{z^3}\frac{\partial z}{\partial x}$$
$$=-\frac{1}{x^2}-\frac{1}{z^2}-\frac{2x^2}{z^4}.$$

But at the stationary point, we have $\frac{\partial V}{\partial x}=0$.

$\therefore$ at the stationary point found above, we have

$$\frac{\partial^2 V}{\partial x^2}=-V\left[\frac{1}{x^2}+\frac{1}{z^2}+\frac{2x^2}{z^4}\right]=-8xyz\left[\frac{1}{x^2}+\frac{1}{z^2}+\frac{2x^2}{z^4}\right],$$

which is $-$ve when $x=y=z=a/\sqrt{3}$.

Thus V is maximum when $x=y=z=a/\sqrt{3}$.

Hence the rectangular solid of maximum volume inscribed in a sphere is a cube.