

INSTITUTE FOR IAS/IFoS/CSIR/GATE EXAMINATIONS
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Any point on the line ① is $(3-\lambda, 8-\lambda, 3+\lambda)$, say point P and any point on the line ② is $(-3-3\lambda', -7+2\lambda', 6+4\lambda')$ say point Q.

Then the direction ratios of the line PQ are

$$(3+\lambda) - (-3-3\lambda'), (8-\lambda) - (-7+2\lambda'), (3+\lambda) - (6+4\lambda')$$

$$3\lambda + 3\lambda' + 6, -\lambda - 2\lambda' + 15, \lambda - 4\lambda' - 3 \quad \text{--- ③}$$

Now if PQ is the S.D between the given lines then PQ is $\perp$ to both ① and ②, the conditions for the same are

$$3(3\lambda + 3\lambda' + 6) - 1(-\lambda - 2\lambda' + 15) + 1(\lambda - 4\lambda' - 3) = 0$$

$$\text{and } -3(3\lambda + 3\lambda' + 6) + 2(-\lambda - 2\lambda' + 15) + 4(\lambda - 4\lambda' - 3) = 0$$

$$\Rightarrow 11\lambda + 7\lambda' = 0 \quad \text{and} \quad 7\lambda + 29\lambda' = 0$$

Solving these we find $\lambda = 0$ and $\lambda' = 0$

Substituting these values of λ and λ' , we find that the coordinates of P and Q are $(3, 8, 3)$ and $(-3, -7, 6)$ respectively.

And the dir's of the line PQ from ③ are $6, 15, -3$ (or) $2, 5, -1$

Now required S.D

$$= PQ = \sqrt{[3 - (-3)]^2 + [8 - (-7)]^2 + [3 - 6]^2}$$

$$= \sqrt{36 + 225 + 9} = 3\sqrt{30}.$$

Also PQ is a line through $P(3, 8, 3)$ and of direction ratios $2, 5, -1$ so its equations are

$$\frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$$