$\Rightarrow a(\cos\theta - \sin\theta) - c(\cos^2\theta - \sin^2\theta) = 0 \quad [\because \delta\theta \neq 0]$

$\Rightarrow (\cos\theta - \sin\theta)[a - c(\cos\theta + \sin\theta)] = 0$

$\therefore \cos\theta - \sin\theta = 0$

i.e, $\sin\theta = \cos\theta \Rightarrow \tan\theta = 1 \Rightarrow \theta = \pi/4$

giving one position of equilibrium in which the lamina rests symmetrically on the pegs.

$\Rightarrow a - c(\cos\theta + \sin\theta) = 0$

$\Rightarrow c^2(\cos\theta + \sin\theta)^2 = a^2$

$\Rightarrow c^2(1 + \sin 2\theta) = a^2$

$\Rightarrow \sin 2\theta = \frac{a^2}{c^2} - 1 = \frac{a^2 - c^2}{c^2}$

$\Rightarrow \theta = \frac{1}{2}\sin^{-1}\left(\frac{a^2 - c^2}{c^2}\right)$

giving the other position of equilibrium.

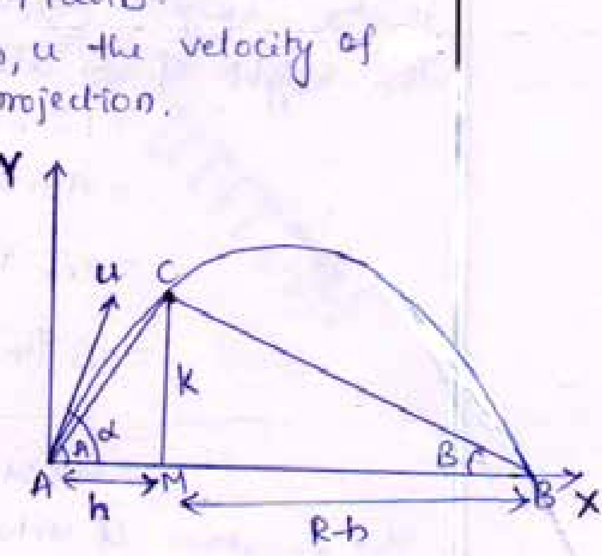
5(d) A particle is thrown over a triangle from one end of a horizontal base and grazing over the vertex falls on the other end of the base. If A, B be the base angles of the triangle and α the angle of projection, Prove that

$$\tan\alpha = \tan A + \tan B.$$

Sol^n: Let A be the point of projection, u the velocity of projection and α the angle of projection.

The particle while grazing over the vertex C falls at the point B.

If $AB = R$, then $R = \frac{2u^2 \sin\alpha\cos\alpha}{g}$ — ①



Take the horizontal line AB as the x-axis and the vertical line AY as the y-axis. Let the coordinates of the vertex C be (h, k). Then the point (h, k) lies on the trajectory whose equation is

$$y = x\tan\alpha - \frac{1}{2}g\frac{x^2}{u^2\cos^2\alpha}$$