then this line being ⊥ar the given three planes, we have $0.l_1 + n.m_1 - m.n_1 = 0$, $-n.l_1 + 0.m_1 + l.n_1 = 0$ and $m.l_1 - n.m_1 + 0.n_1 = 0$

Solving the above three equations in pairs, we get

$$\frac{l_1}{n_1} = \frac{m_1}{mn} = \frac{n_1}{n^2} \Rightarrow \frac{l_1}{l} = \frac{m_1}{m} = \frac{n_1}{n} \quad \text{— (1)}.$$

$$\frac{l_1}{nl} = \frac{m_1}{lm} = \frac{n_1}{n^2} \quad \text{— (2)} \text{ and } \frac{l_1}{-mn} = \frac{m_1}{-m^2} = \frac{n_1}{-nm} \Rightarrow \frac{l_1}{n} \quad \text{— (3)}$$

we have $l = n$ as a condition that the plane intersects in a line. Substituting n for l, all the above three equations (1), (2) and (3) reduce to

$$\frac{l_1}{n} = \frac{m_1}{m} = \frac{n_1}{n} \Rightarrow \frac{l_1}{l} = \frac{m_1}{m} = \frac{n_1}{n} \quad \text{— (4)}$$

Also putting $x = 0$ in the equations of the given planes we have $ny - mz = \lambda$, $lz = \mu$, $-ny = \nu$

these give $x = 0$, $y = -\frac{\nu}{n}$, $z = \frac{\mu}{l}$

and $l\nu + m\mu + l\lambda = 0 \Rightarrow l\lambda + m\mu + n\nu = 0 \quad \because l = n$ — (5)

∴ Any point on the line common to three given planes is $(0, -\nu/n, \mu/l)$

∴ the equations of this common line are

$$\frac{x-0}{l/(\Sigma l^2)} = \frac{y+(\nu/n)}{m/(\Sigma l^2)} = \frac{z-(\mu/l)}{n/(\Sigma l^2)}$$

∴ Square of the length of the ⊥ar from the origin $(0,0,0)$ to the line (5)

$$= \begin{vmatrix} m/\sqrt{\Sigma l^2} & n/\sqrt{\Sigma l^2} \\ 0+(\nu/n) & 0-(\mu/l) \end{vmatrix}^2 + \begin{vmatrix} n/\sqrt{\Sigma l^2} & l/\sqrt{\Sigma l^2} \\ 0-(\mu/l) & 0-0 \end{vmatrix} + \begin{vmatrix} l/\sqrt{\Sigma l^2} & m/\sqrt{\Sigma l^2} \\ 0-0 & 0+\nu/n \end{vmatrix}^2$$

$$= \frac{1}{\Sigma l^2}\left[\begin{vmatrix} m & n \\ \nu/n & -\mu/l \end{vmatrix}^2 + \begin{vmatrix} n & l \\ -\mu/l & 0 \end{vmatrix}^2 + \begin{vmatrix} l & m \\ 0 & \nu/n \end{vmatrix}^2 \right]$$

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