Now suppose that the line segment AB represents the budget line. Along AB $p_1x_1+p_2x_2=M$ holds. Let initial indifference curve of the consumer is IC_0. In IC_0, there are many points along that indifference curve such that $p_1x_1+p_2x_2 \leq M$ holds. Therefore, utility maximising consumer will spend more as she moves to higher indifference curve (say IC_1). In IC_1 there are still such points along the indifference curve such that $p_1x_1+p_2x_2 \leq M$ holds, so again consumer spends more. This process will continue as long as consumer reaches an indifference curve where for no point along the indifference curve $p_1x_1+p_2x_2 \leq M$ holds and at least one point of the indifference curve is on the budget line. At that point, we have consumer equilibrium, $C(x_1, x_2) = (x_1^*(M,p_1,p_2), x_2^*(M,p_1,p_2))$(in Figure 1.5.3 point 'e' is the equilibrium point). Not that at equilibrium, slope of the indifference curve is equal to the slope of the budget line. Therefore, at equilibrium we have

1) Budget constraint holds with equality sign.
2) Slope of the indifference curve is equal to the slope of the budget line.

Mathematical Presentation

Consumer's objective is to maximise her utility by solving UMP. To solve UMP, we set the Lagrange function of the corresponding problem, which is,

$$L(x_1, x_2) = U(x_1, x_2) + \lambda\,(M-p_1x_1-p_2x_2)$$

Our objective is to maximise this Lagrange function by choosing x_1, x_2 and λ. For that we differentiate the Lagrange function by x_1, x_2 and λ, and set all equal to zero.

$$\frac{dL(x_1,x_2)}{dx_1} = \frac{dU(x_1,x_2)}{dx_1} - \lambda p_1 = 0 \text{ ------------- } (f_1)$$

$$\frac{dL(x_1,x_2)}{dx_2} = \frac{dU(x_1,x_2)}{dx_2} - \lambda p_2 = 0 \text{ ------------- } (f_2)$$

$$\frac{dL(x_1,x_2)}{d\lambda} = M - p_1x_1 - p_2x_2 = 0 \text{ ------------- } (f_3)$$

From equation (f1) and (f2), we get,

$\frac{dU(x_1,x_2)}{dx_1} / \frac{dU(x_1,x_2)}{dx_2} = p_1/p_2$. Note $\frac{dU(x_1,x_2)}{dx_1} / \frac{dU(x_1,x_2)}{dx_2}$ is the slope of the indifference curve and p_1/p_2 is the slope of the budget line. So, at equilibrium we have a slope of the indifference curve that is equal to the slope of the budget line. Again, from equation (f3) we get $M = p_1x_1+p_2x_2$, so budget equation holds with equality sign.

Check Your Progress 2

1) Define indifference curve in one sentence. What are measured in the axes of the figure to draw an indifference curve?

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