Because of symmetry, S' is the second focus and the line l' through Z' perpendicular to CZ is the second directrix.

(4) If LSL' is drawn perpendicular to CZ, then LL' is called the latus rectum of the ellipse and its equation is $x = ae$. Solving the equation of the ellipse and the line $x = ae$,

we get $\frac{a^2e^2}{a^2} + \frac{y^2}{b^2} = 1.$

$$\therefore y^2 = b^2(1-e^2) = b^2 \cdot \frac{b^2}{a^2} = \frac{b^4}{a^2}$$

$$\text{i.e., } y = \pm\frac{b^2}{a}.$$

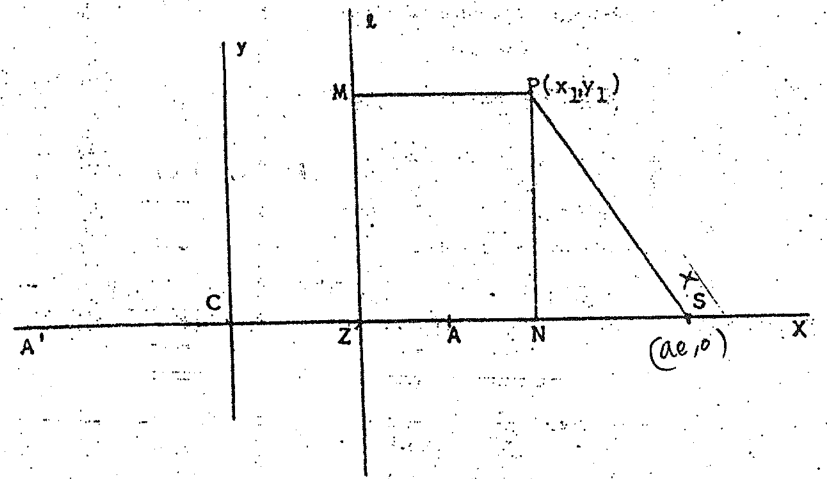
The length of the latus rectum is $\frac{2b^2}{a}$ and the co-ordinates of L are $\left(ae, \frac{b^2}{a}\right)$.

Note: The other standard form of the equation of the ellipse is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$. In this case the major axis is along the y-axis and the minor axis along the x-axis. The foci will have coordinates (o, ae) and $(o, -ae)$.

Article: 3 To derive the standard equation of the Hyperbola in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Let S be the focus and l be the directrix.

Draw SZ perpendicular to l.



Divide SZ internally at A and externally at A' such that

$$\frac{SA}{AZ} = \frac{SA'}{A'Z} = e > 1.$$

Let C be the mid-point of AA' and let $AA' = 2a$. Draw CY perpendicular to CA. Take CX and CY as the x and y-axes.

$$SA = eAZ$$

$$SA' = eZA'$$

$$SA' + SA = e(ZA' + AZ)$$

$$CA' + SC + SC - AC = e(AA')$$

$$2SC = 2ae$$

$$\therefore CS = ae \text{ (in length)}$$

$$SA' - SA = e[ZA' - AZ]$$

$$AA' = e[CA' + ZC - (AC - ZC)]$$

$$2a = e.2CZ$$

$$CZ = \frac{a}{e}.$$

Take any point $P(x_1, y_1)$ on the hyperbola. Draw PM, PN perpendicular to the directrix and x-axis respectively. Then,

$$PM = NZ = CN - CZ = x_1 - \frac{a}{e}$$

Since P is a point on the hyperbola,

$$SP^2 = e^2PM^2$$

$$(x_1 - ae)^2 + y_1^2 = e^2\left[x_1 - \frac{a}{e}\right]^2$$

$$x_1^2 + a^2e^2 - 2x_1ae + y_1^2 = e^2\left[x_1^2 + \frac{a^2}{e^2} - 2x_1\frac{a}{e}\right]$$

$$\text{i.e., } x_1^2 + a^2e^2 - 2x_1ae + y_1^2 = e^2x_1^2 + a^2 - 2x_1ae$$

$$\text{i.e., } a^2(e^2-1) = x_1^2(e^2-1) - y_1^2$$

dividing by $a^2(e^2-1)$, we get

$$\frac{x_1^2}{a^2} - \frac{y_1^2}{a^2(e^2-1)} = 1$$

The locus of (x_1, y_1) is

$$\frac{x_1^2}{a^2} - \frac{y_1^2}{a^2(e^2-1)} = 1.$$